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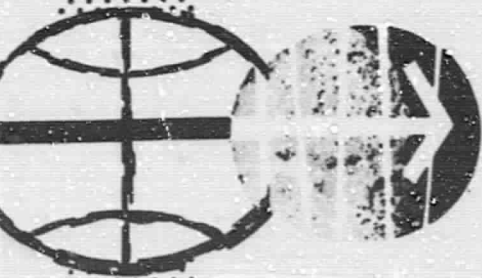
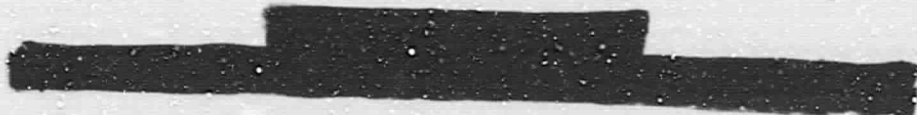
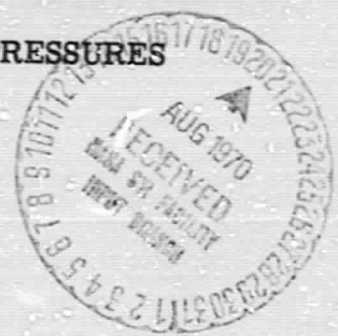
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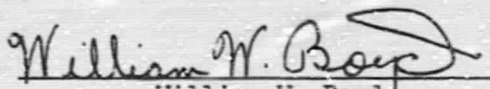
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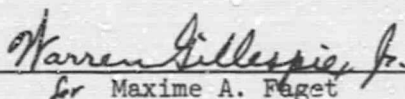
EXCITATION OF FLAT PANELS DUE TO LATERALLY
UNCORRELATED FLUCTUATING PRESSURES

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EXCITATION OF FLAT PANELS DUE TO Laterally Uncorrelated Fluctuating Pressures

By William W. Boyd

SUMMARY

This paper contains an analysis of the response of a panel element to a fluctuating pressure. The pressure is characterized by a lateral correlation coefficient of 0 or 1 and an exponentially damped cosine longitudinal spatial correlation distribution in the direction of wave propagation.

Expressions for the joint acceptance and cross-joint acceptance are developed for this particular forcing function. The results of the analysis are then applied to a flat panel with four edges simply supported.

INTRODUCTION

Various papers have appeared in the literature from time to time which have attempted to clarify the parameters involved in the dynamic response of beams, panels, and other structural elements (refs. 1 and 2). One of the most important of these parameters describes the coupling of an acoustic field with the vibration characteristics of a structure. The value of this parameter, known by acousticians as the "joint acceptance," measures how well the acoustic forcing function fits the structural vibration shape, thereby providing a gauge of the efficiency of the excitation-response coupling.

One of the most interesting and elucidating of the papers concerning the structural acoustic coupling of flat panels was written by Daniel Bozich (ref. 3). Briefly, Bozich discussed the joint acceptance of flat panels to a pressure wave which is characterized by an exponentially damped cosine correlation distribution in the direction of wave propagation. This particular correlation distribution is exhibited in the fluctuating pressure fields created by turbulent airflows around

space vehicles. Correlation measurements taken during wind tunnel tests of rigid body space vehicle models indicate a decrease in spatial correlation which approaches exponential decay as the separation distance between measurement points increases. Bozich chose the longitudinal axis of the panels as the direction of propagation and the pressure was assumed to be uniformly distributed over the panel width, that is, the pressure was perfectly correlated across the width of the panel. As a result of Bozich's assumption regarding the lateral spatial correlation of the pressures, the joint acceptance for all modes with an even number of half-waves across the panel width was zero. Theoretically then, these modes would not contribute to the response of the panel. In reality, the inflight aerodynamic pressure wave would result in some sort of circumferential or lateral correlation distribution over a structure or structural element. However, it is not the purpose of this paper to discuss all of the many possible forms of this lateral correlation distribution but to limit the analysis to a particular type that results from a planned acoustic test condition for large space vehicles. This test consists of enshrouding the periphery of a vehicle by 16 independent acoustic ducts, each duct being fitted to the vehicle in such a way as to prevent the acoustic pressures propagated down it from affecting the pressures in the ducts on either side. No force, however, is applied to the vehicle by the duct assemblies themselves. There is a separate acoustic noise transducer coupled with a random programming signal generator for each of the 16 ducts. With the acoustic ducts, transducers, and signal generators arranged in this fashion, it is possible to obtain very nearly correlated or uncorrelated pressures around the circumference of the structure simply by driving all of the acoustic transducers with a single random signal generator (correlated) or by driving each of the transducers with separate random signal generators (uncorrelated). Although the test will be accomplished on a shell-type structure, an analysis of a flat panel should serve to demonstrate the variation of the joint acceptance coefficients for laterally uncorrelated fluctuating pressures.

It is, therefore, the purpose of this paper to derive expressions for joint acceptance coefficients of a flat panel for the pressure distribution described above. The width of the panel will be assumed to be divided into an arbitrary number of pressure strips, corresponding to the acoustic ducts which span the panel length (fig. 1). Each pressure strip has an exponentially damped cosine correlation distribution over the length of the panel and the lateral correlation coefficient of the pressures between all ducts may be 1 or 0. In order to facilitate a comparison of results presented by Bozich, a flat panel with simply supported edge conditions will be taken as a special case.

SYMBOLS

A	total surface area of panel
A_r, A_q	surface area of panel over which the r^{th} and q^{th} pressure strips are assumed to impinge
a, b	length and width of panel, respectively
D	panel bending rigidity, $\frac{Eh^3}{12(1 - \mu^2)}$
d	number of pressure strips and/or acoustic ducts spanning the width of the panel
E	elastic modulus
$F_{mn}(i\omega), F_{ps}^*(-i\omega)$	$(mn)^{\text{th}}$ and $(ps)^{\text{th}}$ modal component of the Fourier spectrum of the generalized force (*indicates complex conjugate)
$F_{mn}(t), F_{ps}(t)$	$(mn)^{\text{th}}$ and $(ps)^{\text{th}}$ modal components of the generalized force
h	panel thickness
j_{mn}^2	joint acceptance
j_{mnps}^2	cross-joint acceptance
j_{rqmn}^2	r^{th} component of the joint acceptance resulting from the q^{th} pressure strip
j_{rqmnps}	r^{th} component of the cross-joint acceptance resulting from the q^{th} pressure strip
j_{rrmn}^2	r^{th} component of the joint acceptance resulting from the r^{th} pressure strip

j_{rrmnps}^2	r^{th} component of the cross-joint acceptance resulting from the r^{th} pressure strip
K	acoustic wave number (ω/c)
M_{mn}, M_{ps}	generalized mass
m, p	integer mode number associated with the half- bending waves along the longitudinal (x-) axis of the panel
n, s	integer mode number associated with the half- bending waves along the lateral (y-) axis of the panel
$P(x, y, t)$	fluctuating pressure
$P_r(x, y, t), P_q(x, y, t)$	r^{th} and q^{th} fluctuating pressure strips
$R_{rq}(x, x', y, y')$	normalized pressure space correlation for points (x, y) of the r^{th} pressure strip and points (x', y') of the q^{th} pressure strip
$R_{rr}(x, x', y, y')$	normalized pressure space correlation function for points (x, y) and (x', y') measured over the r^{th} pressure strip
r, q	integers associated with an arbitrary pressure strip and/or acoustic duct
$S_{mn}(\omega)$	power density spectrum of the $(mn)^{th}$ modal com- ponent of the generalized force
$S_{mnps}(\omega)$	cross power density spectrum of the $(mnps)^{th}$ modal components of the generalized force
$S_{rq}(z, x', y, y'; \omega)$	pressure space correlation function for points (x, y) of the r^{th} pressure strip and (x', y') of the q^{th} pressure strip

$S_{rq}(\omega)$	cross power density spectrum of the r^{th} and q^{th} pressure strips
$S_{rr}(x, x', y, y; \omega)$	pressure space correlation function for points (x, y) and (x', y') measured over the r^{th} pressure strip
$S_{rr}(\omega)$	power density spectrum of the r^{th} pressure strip
$S_w(\omega)$	power density spectrum of panel displacement
$V_{mn}(s, y), V_{ps}(x, y)$	normalized panel eigenfunctions
W	deflection of flat panel
(x, y)	arbitrary points on surface of panel
(x', y')	arbitrary points, other than (x, y) on surface of panel
Z_{mn}, Z_{ps}	generalized complex mechanical impedance
β	panel damping
λ	acoustic wave length
μ	Poisson's ratio
ρ	panel mass per unit area
ω	circular frequency
Ω, γ, R	integers 1, 2, 3, ...
ϕ_{rq}	degree of correlation between the r^{th} and q^{th} pressure strips

THEORY

The desired expressions for the joint acceptance and cross-joint acceptance coefficients are derived from the solution of the equation of motion of a lightly damped flat panel excited by a homogeneous,

random in space and time, fluctuating pressure field. Based on the assumptions of small-deflection plate theory, the equation of motion of the panel can be written as

$$D\nabla^4 W + \beta \frac{\partial W}{\partial t} + \rho \frac{\partial^2 W}{\partial t^2} = P(x, y, t) \quad (1)$$

The steady-state solution to equation (1) can be expressed in a doubly infinite series

$$W(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{V_{mn}(x, y) F_{mn}(t)}{M_{mn} Z_{mn}} \quad (2)$$

where

$$F_{mn}(t) = \int_A P(x, y, t) V_{mn}(x, y) dA \quad (3)$$

The power density spectrum of the displacement is

$$S_w(\omega) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{V_{mn}^2 S_{mn}(\omega)}{|Z_{mn}|^2 M_{mn}^2} + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \sum_{s=1}^{\infty} \frac{V_{mn} V_{ps} S_{mnps}(\omega)}{Z_{mn} Z_{ps}^* M_{mn} M_{ps}} \quad (4)$$

The power density spectrum and the cross power density spectrum of the generalized force are

$$S_{mn}(\omega) = \lim_{T \rightarrow \infty} \frac{\pi}{T} |F_{mn}|^2 \quad (5)$$

$$S_{mnps}(\omega) = \lim_{T \rightarrow \infty} \frac{\pi}{T} |F_{mn} F_{ps}^*| \quad (6)$$

where F_{mn} is the Fourier spectrum of the generalized force and is defined as

$$F_{mn} = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_{mn}(t) e^{i\omega t} dt$$

and

$$F_{ps} = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_{ps}(t) e^{i\omega t} dt$$

Equations (5) and (6) are easily adapted to the present case. The pressures are assumed to be uniformly distributed over the width of the strip and are characterized by an exponentially damped cosine correlation function along the length of the panel. Thus, for an arbitrary number of pressure strips d there will be d generalized forces of the form $F_{mn}(t)$. Therefore, the total generalized force becomes

$$F_{mn}(t) = \sum_{r=1}^d F_{rmn}(t) \quad (7)$$

where

$$F_{rmn} = \int_{A_r} P_r(x, y, t) V_{mn}(x, y) dA_r$$

Performing the appropriate Fourier transformations on equation (7) and substituting into equations (5) and (6) results in

$$S_{mn} = \int_{A_r} \int_{A_q'} V_{mn}(x, y) V_{mn}(x', y') \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\lim_{T \rightarrow \infty} \frac{1}{2T} \right. \right. \\ \left. \left. \int_{-T}^T \sum_{r=1}^d \sum_{q=1}^d P_r P_q dt \right] e^{i\omega\tau} d\tau \right\} dA_r dA_q' \quad (8)$$

$$S_{mrps} = \int_{A_r} \int_{A_q'} V_{mn}(x, y) V_{ps}(x', y') \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\lim_{T \rightarrow \infty} \frac{1}{2T} \right. \right. \\ \left. \left. \int_{-T}^T \sum_{r=1}^d \sum_{q=1}^d P_r P_q dt \right] e^{i\omega\tau} d\tau \right\} dA_r dA_q' \quad (9)$$

Separating equations (8) and (9) into the sums of two series

$$S_{mn} = \int_{A_r} \int_{A_r'} V_{mn} V_{mn}' \sum_{r=1}^d S_{rr}(x, x', y, y'; \omega) dA_r dA_r' + \int_{A_r} \int_{A_q'} V_{mn} V_{mn}' \sum_{r=1}^d \sum_{q=1}^d S_{rq}(x, x', y, y'; \omega) dA_r dA_q' (r \neq q) \quad (10)$$

$$S_{mnps} = \int_{A_r} \int_{A_r'} V_{mn} V_{ps}' \sum_{r=1}^d S_{rr}(x, x', y, y'; \omega) dA_r dA_r' + \int_{A_r} \int_{A_q'} V_{mn} V_{ps}' \sum_{r=1}^d \sum_{q=1}^d S_{rq}(x, x', y, y'; \omega) dA_r dA_q' (r \neq q) \quad (11)$$

Rewriting equations (10) and (11)

$$S_{mn} = A^2 \sum_{r=1}^d \frac{S_{rr}}{A^2} \int_{A_r} \int_{A_r'} V_{mn} V_{mn}' \frac{S_{rr}(x, x', y, y'; \omega)}{S_{rr}(\omega)} dA_r dA_r' + A^2 \sum_{r=1}^d \sum_{q=1}^d \frac{S_{rq}}{A^2} \int_{A_r} \int_{A_q'} V_{mn} V_{mn}' \frac{S_{rq}(x, x', y, y'; \omega)}{S_{rq}(\omega)} dA_r dA_q' (r \neq q) \quad (12)$$

$$\begin{aligned}
S_{mnps} = & A^2 \sum_{r=1}^d \frac{S_{rr}}{A^2} \iint_{A_r A_r'} V_{mn} V_{ps} \frac{S_{rr}(x, x', y, y'; \omega)}{S_{rr}(\omega)} dA_r dA_r' \\
& + A^2 \sum_{r=1}^d \sum_{q=1}^d \frac{S_{rq}}{A^2} \iint_{A_r A_q'} V_{mn} V_{ps} \frac{S_{rq}(x, x', y, y'; \omega)}{S_{rq}(\omega)} dA_r dA_q' (r \neq q)
\end{aligned} \tag{13}$$

Equations (12) and (13) can then be written as

$$\begin{aligned}
S_{mn} = & A^2 \sum_{r=1}^d S_{rr} \cdot \frac{1}{A^2} \iint_{A_r A_r'} R_{rr}(x, x', y, y') V_{mn} V_{mn}' dA_r dA_r' \\
& + A^2 \sum_{r=1}^d \sum_{q=1}^d S_{rq} \cdot \frac{1}{A^2} \iint_{A_r A_q'} R_{rq}(x, x', y, y') V_{mn} V_{mn}' dA_r dA_q' (r \neq q)
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
S_{mnps} = & A^2 \sum_{r=1}^d S_{rr} \cdot \frac{1}{A^2} \iint_{A_r A_r'} R_{rr}(x, x', y, y') V_{mn} V_{ps}' dA_r dA_r' \\
& + A^2 \sum_{r=1}^d \sum_{q=1}^d S_{rq} \cdot \frac{1}{A^2} \iint_{A_r A_q'} R_{rq}(x, x', y, y') V_{mn} V_{ps}' dA_r dA_q' (r \neq q)
\end{aligned} \tag{15}$$

where

$$R_{rr}(x, x', y, y') = \frac{S_{rr}(x, x', y, y'; \omega)}{S_{rr}(\omega)}$$

and

$$R_{rq}(x, x', y, y') = \frac{S_{rq}(x, x', y, y'; \omega)}{S_{rq}(\omega)}$$

Finally equations (14) and (15) become

$$S_{mn} = A^2 \sum_{r=1}^d S_{rr}(\omega) j_{rrmn}^2(\omega) + A^2 \sum_{r=1}^d \sum_{q=1}^d S_{rq}(\omega) j_{rqmn}^2(\omega) (r \neq q) \quad (16)$$

and

$$S_{mnps} = A^2 \sum_{r=1}^d S_{rr}(\omega) j_{rrmnps}^2(\omega) + A^2 \sum_{r=1}^d \sum_{q=1}^d S_{rq}(\omega) j_{rqmnps}^2(\omega) (r \neq q) \quad (17)$$

where

$$j_{rrmn}^2 = \frac{1}{A^2} \int_{A_r} \int_{A_r'} R_{rr}(x, x', y, y') V_{mn} V_{mn}' dA_r dA_r' \quad (18)$$

$$j_{rqmn}^2 = \frac{1}{A^2} \int_{A_r} \int_{A_q'} R_{rq}(x, x', y, y') V_{mn} V_{mn}' dA_r dA_q' \quad (19)$$

$$j_{rrmnps}^2(\omega) = \frac{1}{A^2} \int_{A_r} \int_{A_r'} R_{rr}(x, x', y, y') V_{mn} V_{ps}' dA_r dA_r' \quad (20)$$

$$j_{rqmnps}^2(\omega) = \frac{1}{A^2} \int_{A_r} \int_{A_q'} R_{rq}(x, x', y, y') V_{mn} V_{ps}' dA_r dA_q' (r \neq q) \quad (21)$$

Now, let $S_{rr}(\omega) = S(\omega)$ and $S_{rq}(\omega) = S(\omega) \cos \phi_{rq}$ where $\cos \phi_{rq}$ represents the degree of correlation between the r^{th} pressure strip and the q^{th} pressure strip. Equations (16) and (17) can be written as

$$S_{mn} = A^2 S(\omega) \left[\sum_{r=1}^d j_{rrmn}^2 + \sum_{r=1}^d \sum_{q=1}^d j_{rqmn}^2 \cos \phi_{rq} \right] \quad (22)$$

and

$$S_{mnps} = A^2 S(\omega) \left[\sum_{r=1}^d j_{rrmnps}^2 + \sum_{r=1}^d \sum_{q=1}^d j_{rqmnps}^2 \cos \phi_{rq} \right] \quad (23)$$

The bracketed terms in equations (22) and (23) are the desired expressions for the joint acceptance and cross-joint acceptance

$$j_{mn}^2 = \sum_{r=1}^d j_{rrmn}^2 + \sum_{r=1}^d \sum_{q=1}^d j_{rqmn}^2 \cos \phi_{rq} \quad (24)$$

$$j_{mnps}^2 = \sum_{r=1}^d j_{rrmnps}^2 + \sum_{r=1}^d \sum_{q=1}^d j_{rqmnps}^2 \cos \phi_{rq} \quad (25)$$

Substituting equations (22) and (23) into equation (4), the displacement power density spectrum of the panel becomes

$$S_w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{A^2 V_{mn}^2 S(\omega) j_{mn}^2}{|Z_{mn}|^2 M_{mn}^2} + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \sum_{s=1}^{\infty} \frac{A^2 V_{mn} V_{ps} S(\omega) j_{mnps}^2}{Z_{mn} Z_{ps}^* M_{mn} M_{ps}} \quad (26)$$

Let us examine equations (24) and (25) in terms of the pressure distribution described above, rewriting the equations in the following way

$$j_{mn}^2 = \frac{1}{A^2} \sum_{r=1}^d \int_{A_r} \int_{A_r'} R_{rr}(x, x', y, y') V_{mn} V_{mn}' dA_r dA_r' \\ + \frac{1}{A^2} \sum_{r=1}^d \sum_{q=1}^d \int_{A_r} \int_{A_q'} R_{rq}(x, x', y, y') \cos \phi_{rq} V_{mn} V_{mn}' dA_r dA_q' \quad (27)$$

$$j_{mnps}^2 = \frac{1}{A^2} \sum_{r=1}^d \int_{A_r} \int_{A_r'} R_{rr}(x, x', y, y') V_{mn} V_{ps}' dA_r dA_r' \\ + \frac{1}{A^2} \sum_{r=1}^d \sum_{q=1}^d \int_{A_r} \int_{A_q'} R_{rq}(x, x', y, y') \cos \phi_{rq} V_{mn} V_{ps}' dA_r dA_q' \quad (28)$$

The functions $R_{rr}(x, x', y, y')$ and $R_{rq}(x, x', y, y')$ in equations (27) and (28) are the spatial correlation distributions which, in the case of aerodynamically generated pressure waves propagating along the X-axis of the panel, are approximately a damped cosine function, that is

$$R_{rr}(x, x', y, y') = R_{rq}(x, x', y, y') = e^{-PK(x-x')} \cos K(x - x') \quad (29)$$

Substituting equation (29) into equations (27) and (28) gives

$$\begin{aligned} j_{mn}^2 = & \sum_{r=1}^d \frac{1}{A^2} \int_{A_r} \int_{A_r} V_{mn} V_{mn}' e^{-PK(x-x')} \cos K(x - x') dA_r dA_r' \\ & + \sum_{r=1}^d \sum_{q=1}^d \frac{1}{A^2} \int_{A_r} \int_{A_q} V_{mn} V_{mn}' e^{-PK(x-x')} \cos K(x - x') \cos \phi_{rq} dA_r dA_q' \end{aligned} \quad (30)$$

$$\begin{aligned} j_{mnps}^2 = & \sum_{r=1}^d \frac{1}{A^2} \int_{A_r} \int_{A_r} V_{mn} V_{ps}' e^{-PK(x-x')} \cos K(x - x') dA_r dA_r' \\ & + \sum_{r=1}^d \sum_{q=1}^d \frac{1}{A^2} \int_{A_r} \int_{A_q} V_{mn} V_{ps}' e^{-PK(x-x')} \cos K(x - x') \cos \phi_{rq} dA_r dA_q' \end{aligned} \quad (31)$$

It can be assumed that the eigenfunctions $V_{mn}(x,y)$ can be expressed as a product of two eigenfunctions for beams having end conditions similar to the panel. Thus

$$V_{mn}(x,y) = \phi_m(x)\psi_n(y) \quad (32)$$

Substituting equation (32) into equations (30) and (31)

$$\begin{aligned} j_{mn}^2 = & \sum_{r=1}^d \frac{1}{A^2} \int_{A_r} \int_{A_r'} \psi_n(y) \psi_n(y') \phi_m(x) \phi_m(x') e^{-PK(x-x')} \\ & \cos K(x-x') dA_r dA_r' + \sum_{r=1}^d \sum_{q=1}^d \frac{1}{A^2} \\ & \int_{A_r} \int_{A_q'} \psi_n(y) \psi_n(y') \phi_m(x) \phi_m(x') e^{-PK(x-x')} \\ & \cos K(x-x') \cos \phi_{rq} dA_r dA_q' \end{aligned} \quad (33)$$

$$\begin{aligned} j_{mnps}^2 = & \sum_{r=1}^d \frac{1}{A^2} \int_{A_r} \int_{A_r'} \psi_n(y) \psi_p(y') \phi_m(x) \phi_s(x') e^{-PK(x-x')} \\ & \cos K(x-x') dA_r dA_r' + \sum_{r=1}^d \sum_{q=1}^d \frac{1}{A^2} \\ & \int_{A_r} \int_{A_q'} \psi_n(y) \psi_p(y') \phi_m(x) \phi_s(x') e^{-PK(x-x')} \\ & \cos K(x-x') \cos \phi_{rq} dA_r dA_q' \end{aligned} \quad (34)$$

Now let us define

$$\Gamma_{mm}(\omega) = \frac{1}{a^2} \int_0^a \int_0^a \phi_m(x) \phi_m(x') e^{-PK(x-x')} \cos K(x-x') dx dx' \quad (35)$$

$$\Gamma_{ms}(\omega) = \frac{1}{a^2} \int_0^a \int_0^a \phi_m(x) \phi_s(x') e^{-PK(x-x')} \cos K(x-x') dx dx' \quad (36)$$

And further

$$\theta_{nnrr} = \frac{1}{b^2} \int_{Y_{r-1}}^{Y_r} \int_{Y_{r-1}}^{Y_r} \psi_n(y) \psi_n(y') dy dy' \quad (37)$$

$$\theta_{nnrq} = \frac{1}{b^2} \int_{Y_{r-1}}^{Y_r} \int_{Y_{q-1}}^{Y_q} \psi_n(y) \psi_n(y') dy dy' \quad (38)$$

$$\theta_{nprr} = \frac{1}{b^2} \int_{Y_{r-1}}^{Y_r} \int_{Y_{r-1}}^{Y_r} \psi_n(y) \psi_p(y') dy dy' \quad (39)$$

$$\theta_{nprq} = \frac{1}{b^2} \int_{Y_{r-1}}^{Y_r} \int_{Y_{q-1}}^{Y_q} \psi_n(y) \psi_p(y') dy dy' \quad (40)$$

Equations (33) and (34) then can be written

$$j_{mn}^2(\omega) = \sum_{r=1}^d \theta_{nnrr} \Gamma_{mm}(\omega) + \sum_{r=1}^d \sum_{q=1}^d \theta_{nnrq} \Gamma_{mm}(\omega) \cos \phi_{rq} \quad (41)$$

$$j_{mnps}^2(\omega) = \sum_{r=1}^d \theta_{npr r} \Gamma_{ms}(\omega) + \sum_{r=1}^d \sum_{q=1}^d \theta_{npr q} \Gamma_{ms}(\omega) \cos \phi_{rq} \quad (42)$$

The bounds on the lateral correlation ϕ_{rq} are:

1. Each pressure strip is generated with an independent programming signal source, that is, $\cos \phi_{rq} = 0$.

2. Each pressure strip is generated with the same programming signal source, that is, $\cos \phi_{rq} = 1$.

The case for which $\cos \phi_{rq} = 1$ will be discussed later since it reduces to the correlation conditions in Bozich's analysis. For now, it is most interesting to examine the condition for which $\cos \phi_{rq} = 0$, that is, the r^{th} pressure strip is completely uncorrelated with the q^{th} pressure strip. Equations (41) and (42) then reduce to

$$j_{mn}^2 = \sum_{r=1}^d \theta_{nnrr} \Gamma_{mm}(\omega) \quad (43)$$

$$j_{mnps}^2 = \sum_{r=1}^d \theta_{npr r} \Gamma_{ms}(\omega) \quad (44)$$

Thus, we have an expression for the joint acceptance and cross-joint acceptance of a panel excited by laterally uncorrelated acoustic pressures.

SPECIAL CASE

It is helpful at this point to discuss a specific example so as to determine how the laterally uncorrelated pressures will affect the panel response. For simplicity, a flat panel which has simple supported edge conditions has been chosen. The panel eigenfunctions in this case are

$$V_{mn}(x,y) = \phi_m(x)\psi_n(y) = \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (45)$$

Rewriting equations (37) and (39) and integrating over the limits $y_{r-1} = \frac{(r-1)b}{d}$ and $y_r = \frac{rb}{d}$, we obtain

$$\theta_{nnrr} = \frac{1}{b^2} \int_{\frac{(r-1)b}{d}}^{\frac{rb}{d}} \int_{\frac{(r-1)b}{d}}^{\frac{rb}{d}} \sin \frac{n\pi y}{b} \sin \frac{n\pi y'}{b} dy dy' \quad (46)$$

and

$$\theta_{nprr} = \frac{1}{b^2} \int_{\frac{(r-1)b}{d}}^{\frac{rb}{d}} \int_{\frac{(r-1)b}{d}}^{\frac{rb}{d}} \sin \frac{n\pi y}{b} \sin \frac{p\pi y'}{b} dy dy' \quad (47)$$

Integrating, equations (46) and (47) reduce to

$$\theta_{nnrr} = \frac{1}{(n\pi)^2} \left[\cos \frac{rn\pi}{d} - \cos \frac{(r-1)n\pi}{d} \right]^2 \quad (48)$$

and

$$\theta_{nprr} = \frac{1}{np\pi^2} \left[\cos \frac{rn\pi}{d} - \cos \frac{(r-1)n\pi}{d} \right] \left[\cos \frac{rp\pi}{d} - \cos \frac{(r-1)p\pi}{d} \right] \quad (49)$$

Substitution of equations (48) and (49) into equations (43) and (44) yields the expressions

$$j_{mn}^2 = \frac{1}{(n\pi)^2} \sum_{r=1}^d \left[\cos \frac{rn\pi}{d} - \cos \frac{(r-1)n\pi}{d} \right]^2 \Gamma_{mm}(\omega) \quad (50)$$

$$j_{mnps}^2 = \frac{1}{np\pi^2} \sum_{r=1}^d \left[\cos \frac{rn\pi}{d} - \cos \frac{(r-1)n\pi}{d} \right] \left[\cos \frac{rp\pi}{d} - \cos \frac{(r-1)p\pi}{d} \right] \Gamma_{ms}(\omega) \quad (51)$$

Rewriting equations (50) and (51)

$$j_{mn}^2 = \frac{4d}{(n\pi)^2} \Delta_{nn} \Gamma_{mm}(\omega) \quad (52)$$

$$j_{mnps}^2 = \frac{4d}{np\pi^2} \Delta_{np} \Gamma_{ms}(\omega) \quad (53)$$

where

$$\Delta_{nn} = \frac{1}{4d} \sum_{r=1}^d \left[\cos \frac{rn\pi}{d} - \cos \frac{(r-1)n\pi}{d} \right]^2 \quad (54)$$

$$\Delta_{np} = \frac{1}{4d} \sum_{r=1}^d \left[\cos \frac{rn\pi}{d} - \cos \frac{(r-1)n\pi}{d} \right] \left[\cos \frac{rp\pi}{d} - \cos \frac{(r-1)p\pi}{d} \right] \quad (55)$$

At this point the cross-joint acceptance, as expressed by equation (53), will be neglected in order to use the joint acceptance equation (52) to make a direct comparison with Bozich's results. Upon integrating equation (33), the expression for $\Gamma_{mm}(\omega)$ becomes

$$\begin{aligned} \Gamma_{mm}(\omega) = \frac{1}{a^2} & \left\{ e^{-\chi a} \left[\left(\frac{H}{\chi^2 + H^2} + \frac{I}{\chi^2 + I^2} \right) \left(\frac{[-\chi \sin Ha - H \cos Ha]}{H^2 + \chi^2} \right. \right. \right. \\ & + \left. \left. \frac{[\chi \sin Ia - I \sin Ha]}{\chi^2 + I^2} \right) - \left(\frac{\chi}{\chi^2 + I^2} - \frac{\chi}{\chi^2 + H^2} \right) \right] \\ & \cdot \left[\left(\frac{[I \sin Ia - \chi \cos Ia]}{\chi^2 + I^2} \right) - \left(\frac{[H \sin Ha - \chi \cos Ha]}{\chi^2 + H^2} \right) \right. \\ & + \left. \frac{a}{2} \left(\frac{\chi}{\chi^2 + I^2} - \frac{\chi}{\chi^2 + H^2} \right) + \frac{a}{2} \left(\frac{I}{\chi^2 + I^2} + \frac{H}{\chi^2 + H^2} \right) \right. \\ & \left. \left. - \frac{a}{2} \left(\frac{\chi}{\chi^2 + I^2} - \frac{\chi}{\chi^2 + H^2} \right) \right] \right\} \quad (56) \end{aligned}$$

where

$$\chi = \frac{2p\pi}{\lambda}, \quad H = \left(\frac{m\pi}{a} + K \right), \quad \text{and} \quad I = \left(\frac{m\pi}{a} - K \right)$$

It should be observed that for an equivalent decay constant P equation (56) would exactly correspond to Bozich's equation (19) (ref. 4).

RESULTS AND DISCUSSION

The analysis of the response of a flat panel due to the strip loading of laterally uncorrelated fluctuating pressures has resulted in expressions for the joint acceptance and cross-joint acceptance coefficients which are applicable to all panels with classical boundary conditions. However, to facilitate making a comparison with an earlier analysis by Bozich, a panel with simply supported edges was taken as a special case.

The laterally correlated pressures assumed by Bozich resulted in zero joint acceptance coefficients for all modes having an even number of half-wave lengths along the lateral axis of the simply supported panel. However, the analysis of a similarly supported panel for the laterally uncorrelated pressures produced results quite to the contrary.

The essential difference in the two joint acceptance coefficients lies in the function Δ_{nn} defined by equation (54). Figure 2 was constructed to demonstrate the dependence of Δ_{nn} on the number of pressure strips and the mode number. It should be noticed that the function Δ_{nn} is zero for n/d ratios which are even integers. In fact, an interesting relationship can be derived by evaluating this function at the zero point

$$\Delta_{nn} = \frac{1}{4d} \sum_{r=1}^d \left[\cos \frac{rn\pi}{d} - \cos \frac{(r-1)n\pi}{d} \right]^2 = 0 \quad (57)$$

Since the bracketed term of equation (57) is always positive, every term in the summation must be identically zero. So the V^{th} term may be written

$$\cos \frac{V\pi n}{d} = \cos \frac{(V-1)n\pi}{d}; \quad V = 1, 2, 3, \dots \quad (58)$$

where

$$\frac{Vn}{d} = \gamma; \quad \gamma = 1, 2, 3, \dots \quad (59)$$

and

$$\frac{(V-1)n}{d} = \Omega; \quad \Omega = 1, 2, 3, \dots \quad (60)$$

Substituting equation (59) into equation (60)

$$(\gamma - \Omega) = \frac{n}{d} \quad (61)$$

Equation (58) indicates that γ and Ω must either simultaneously be odd integers or even integers. As a result of this it is easy to see that the difference between γ and Ω will be an integer multiple of 2, that is

$$(\gamma - \Omega) = 2R = \frac{n}{d}; \quad R = 1, 2, 3, \dots \quad (62)$$

Thus, for a specific number of pressure strips, all joint acceptance coefficients will have a value other than zero except those for which the lateral mode number n is equal to the number of pressure strips times $2R$, $R = 1, 2, 3, 4, \dots$. Interpreting these results in still another way, the zero joint acceptance coefficients occur whenever an even number of lateral half-waves coincide with the width of a pressure strip. Remembering that Bozich's analysis is equivalent to an arbitrary number of correlated pressure strips, equation (41) can be

used (with $\cos \theta_{rq} = 1$) to emphasize the differences in the results for the two pressure correlation conditions

$$j_{mn}^2 = \left[\sum_{r=1}^d \theta_{mnrr} + \sum_{r=1}^d \sum_{q=1}^d \theta_{nnrq} \right] \Gamma_{mm} \quad (63)$$

Rewriting equation (63)

$$j_{mn}^2 = \frac{4}{(n\pi)^2} \Delta_{nn}' \Gamma_{mm} \quad (64)$$

where

$$\Delta_{nn}' = \frac{1}{4} \left\{ \sum_{r=1}^d \left[\cos \frac{rn\pi}{d} - \cos \frac{(r-1)n\pi}{d} \right]^2 + \sum_{r=1}^d \sum_{q=1}^d \left[\cos \frac{rn\pi}{d} - \cos \frac{(r-1)n\pi}{d} \right] \left[\cos \frac{qn\pi}{d} - \cos \frac{(q-1)n\pi}{d} \right] \right\} \quad (65)$$

$$\Delta_{nn}' = \begin{cases} 1 & \text{for } n = 1, 3, 5, \dots, 2n-1 \\ 0 & \text{for } n = 2, 4, 6, \dots, 2n \end{cases}$$

Figures 3 and 4 illustrate the values of the two functions Δ_{nn} and Δ_{nn}' for several mode numbers. Figure 3 represents the case for which three pressure strips occur over the panel width. As seen in equation (65) above, Bozich's analysis indicates that the joint acceptance coefficients for $n = 2, 4, 6, 8, 10$, et cetera, would be zero, while the odd modes $n = 1, 3, 5$, et cetera, would have nonzero coefficients. The uncorrelated pressure strips, however, produce nonzero joint acceptance

coefficients for the $n = 2, 4, 8, 10$ modes in addition to all of the odd modes $n = 1, 3, 5$, et cetera. As indicated earlier, the lateral modes which have zero joint acceptance coefficients are $n = 6, 12$, that is, $2Rd$. Figure 4 shows the number of modes having nonzero joint acceptance coefficients for the condition where four pressure strips are distributed over the width of the panel. Again, the case of correlated pressure strips, corresponding to Bozich's analysis, indicates that only the odd number modes have nonzero joint acceptance coefficients. The uncorrelated pressure strips, however, result in nonzero joint acceptance coefficients for the $n = 2, 4, 6, 10, 12$ modes as well as all odd modes $n = 1, 3, 5, 7$, et cetera, and as expected, the mode which results in eight lateral antinodes has a joint acceptance of zero.

An interesting comparison of the joint acceptance coefficient frequency spectra of two modes for laterally uncorrelated and correlated pressure strips is shown in figure 5. The figure was constructed for the specific case of 16 pressure strips distributed over the panel width. It is interesting to note that for the particular modes shown in figure 5 the laterally uncorrelated pressure strips resulted in joint acceptance coefficients which are only slightly lower than the coefficients for the laterally correlated pressure strips. It should also be observed that the shape of the coefficient frequency spectra for laterally uncorrelated pressures is similar in shape to the corresponding coefficient frequency spectra for the laterally correlated pressures. Indeed, it is easily demonstrated that only a factor involving the width of the pressure strips and Δ_{nn} separates the two cases. Bozich's expression for the joint acceptance can be written as

$$j_{mn}^2 \Big|_B = \frac{4}{(n\pi)^2} \Gamma_{mn}(\omega) \quad (66)$$

Dividing equation (52) by equation (66)

$$\frac{j_{mn}^2}{j_{mn}^2 \Big|_B} = d\Delta_{nn}; \quad n = 1, 3, 5, \dots, 2n - 1 (d \neq 1) \quad (67)$$

It is easily seen that for a given number of pressure strips the ratio $j_{mn}^2 / j_{mn}^2_B$ approaches a maximum of d as Δ_{nn} approaches 1 which occurs for values of $\frac{n}{d} = 1, 3, 5, 7$, et cetera. This simply means that the more closely an odd number of half-waves n coincides with the width of a pressure strip the larger will be the ratio $j_{mn}^2 / j_{mn}^2_B$.

The dependence of this ratio on Δ_{nn} is substantiated by figure 5. Figure 2 shows that Δ_{nn} is very small in the region where $\frac{n}{d}$ is very small, and thus, the ratio of the joint acceptance coefficient frequency spectra for the two correlation conditions would be expected to be small, as verified by figure 5. It can be anticipated, therefore, that a comparison between a laterally uncorrelated and a laterally correlated fluctuating pressure field would reveal that the uncorrelated pressures were less effective in exciting the lower vibration modes than the correlated pressure field. However, the laterally correlated pressure field would tend to suppress the modes with an even number of lateral half-waves.

Figure 6 shows the modes having nonzero joint acceptance coefficients for the cases of 16 pressure strips which are uncorrelated and completely correlated. The correlated pressure strips have zero joint acceptance coefficients for all even lateral mode numbers while all of the odd mode numbers have nonzero coefficients. The uncorrelated pressure strips result in nonzero joint acceptance coefficients for all modes except those which have $2Rd$ lateral antinodes, such as $n = 32$.

Figures 7 and 8 are included to illustrate the shape of the joint acceptance frequency spectra for several modes of vibration which have an even number of lateral antinodes. These figures were constructed for the case where 16 pressure strips are distributed over the panel width. Figures 7 and 8 also serve to demonstrate the selectivity of these panel modes for decay factors of $P = 1/20$ and $P = 1$, respectively.

CONCLUDING REMARKS

The mathematical analysis of the response of a flat panel to an arbitrary number of pressure strips has resulted in expressions for the joint acceptance and cross-joint acceptance which can be evaluated for various lateral correlation conditions between the pressure strips. Applying the results of the analysis to the special case of a simply

supported panel excited by completely uncorrelated pressure strips revealed a definite dependence of the joint acceptance coefficients upon the number of pressure strips and the lateral mode number. It was found that all joint acceptance coefficients involving the odd lateral mode numbers were greater than zero. With respect to the joint acceptance coefficients involving the even lateral mode numbers, however, all were nonzero except those for which an even number of half-waves coincided with the width of a pressure strip.

In addition, an interesting relationship was found to exist between the joint acceptance coefficients resulting from laterally uncorrelated pressure strips and the coefficients resulting from laterally correlated pressure strips. The ratio of the uncorrelated to correlated joint acceptance coefficients proved to be very small for the lower lateral modes. An experiment involving the response of a structure to both a correlated and an uncorrelated pressure field might show that the correlated pressure field produces higher responses in the frequency regime corresponding to the lower lateral modes.

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4. See reference 3, p. 55.

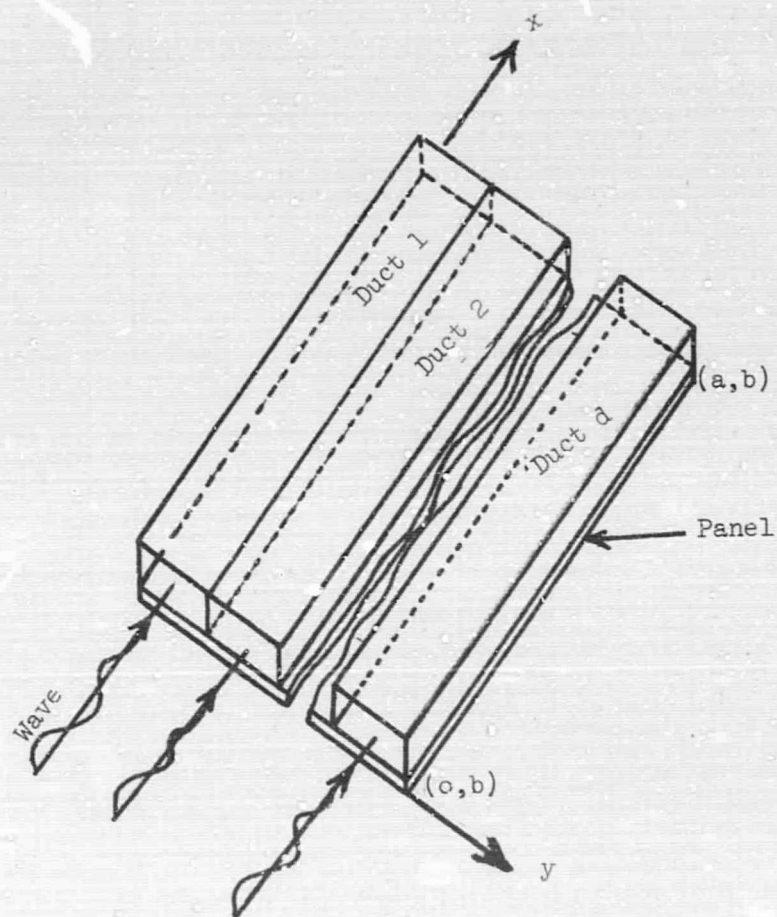


Figure 1.- Panel covered by an arbitrary number of acoustic ducts.

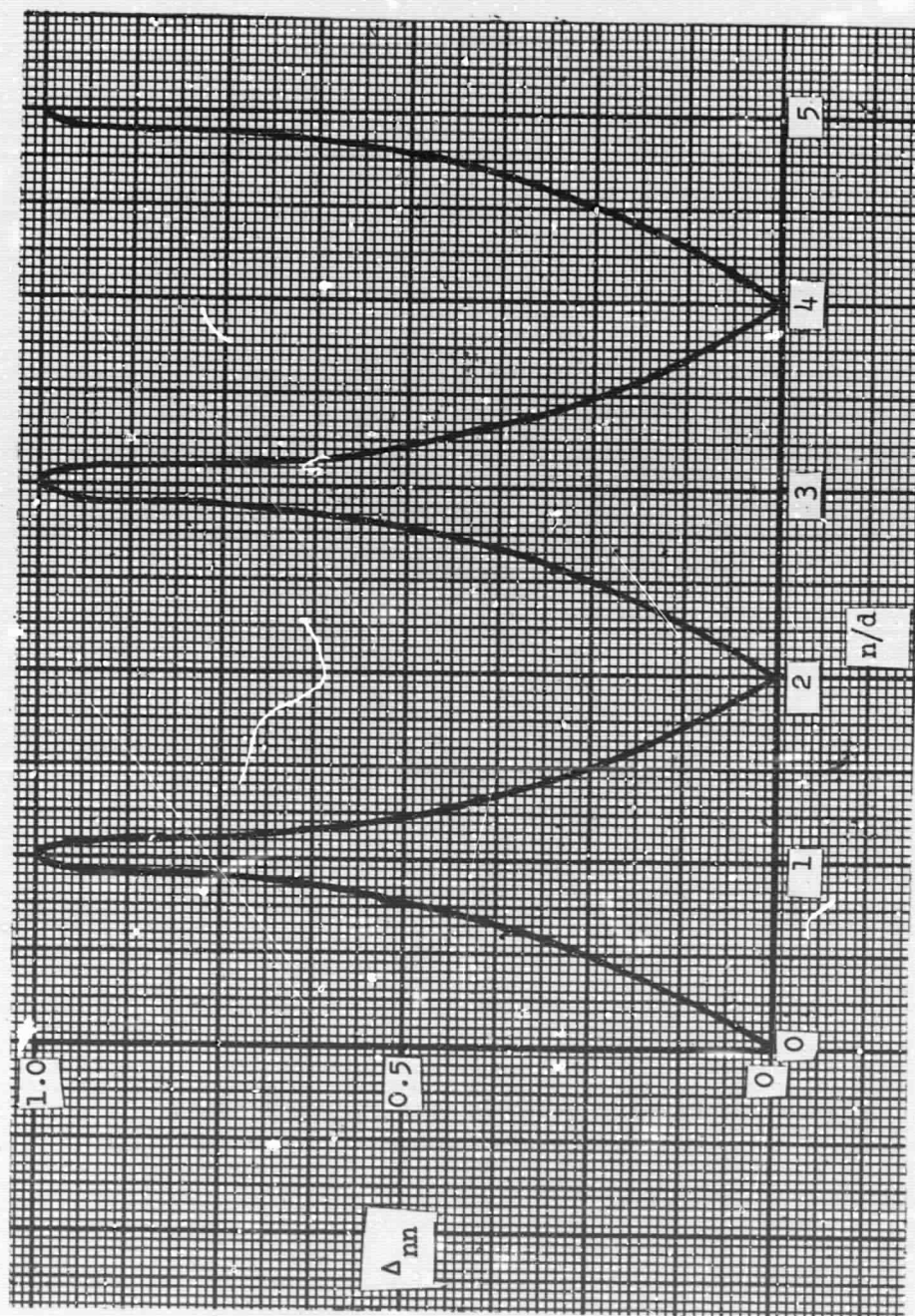


Figure 2.- Variation in joint acceptance coefficients with n/d .

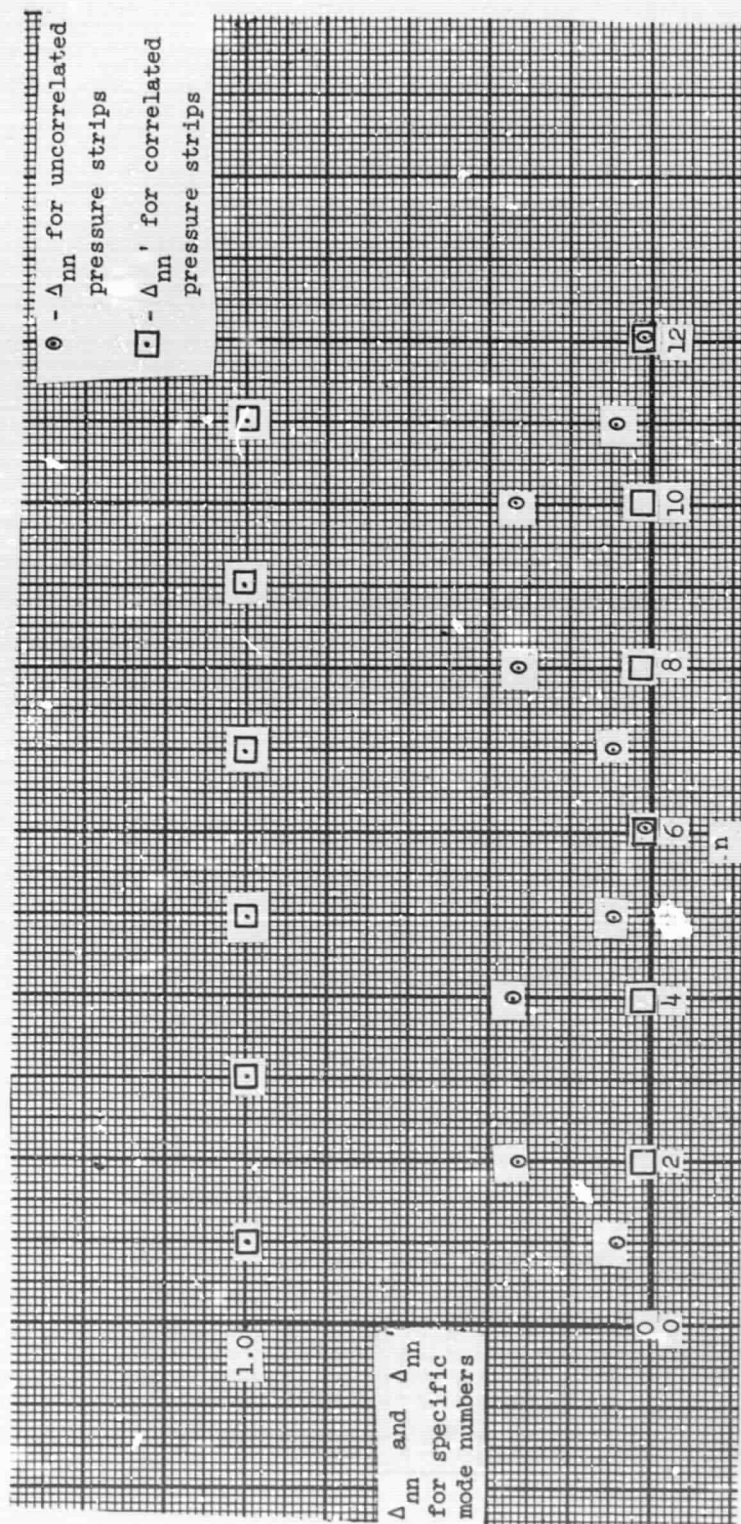


Figure 3.- Comparison of Δ_{nn} and Δ'_{nn} for $d = 3$.

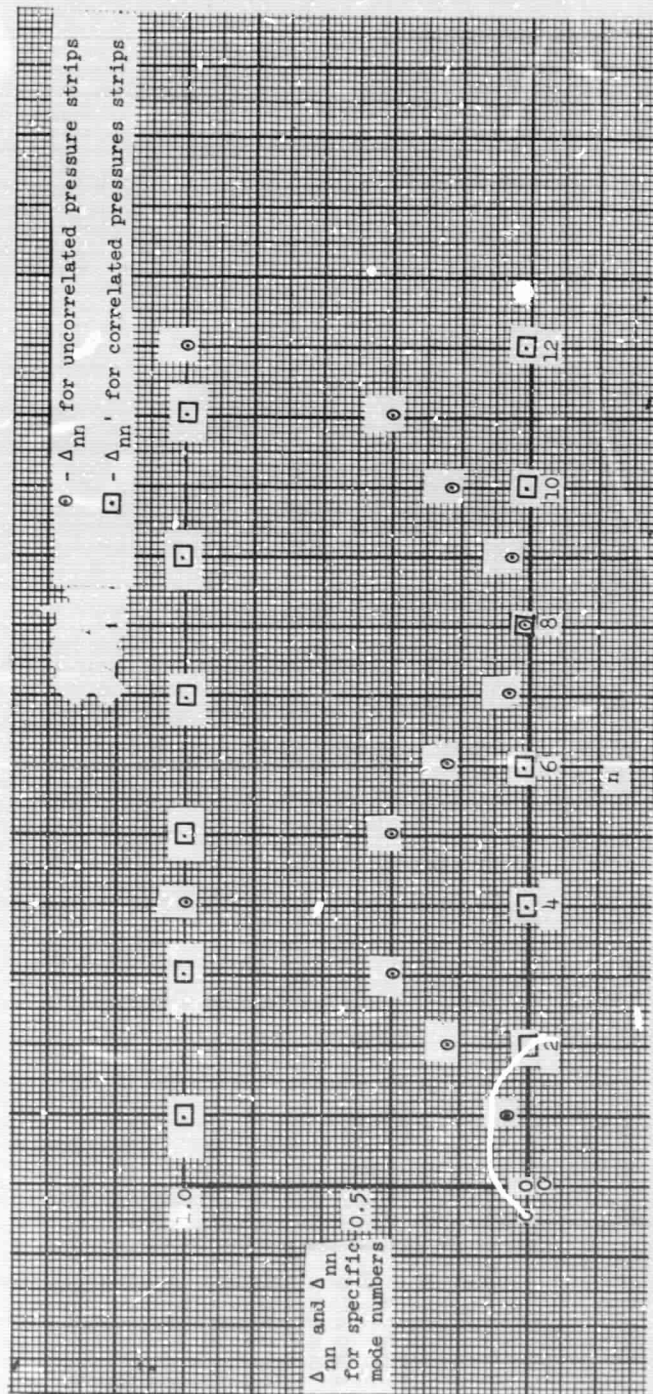


Figure 4.- Comparison of Δ_{nn} and Δ'_{nn} for $d = 4$.

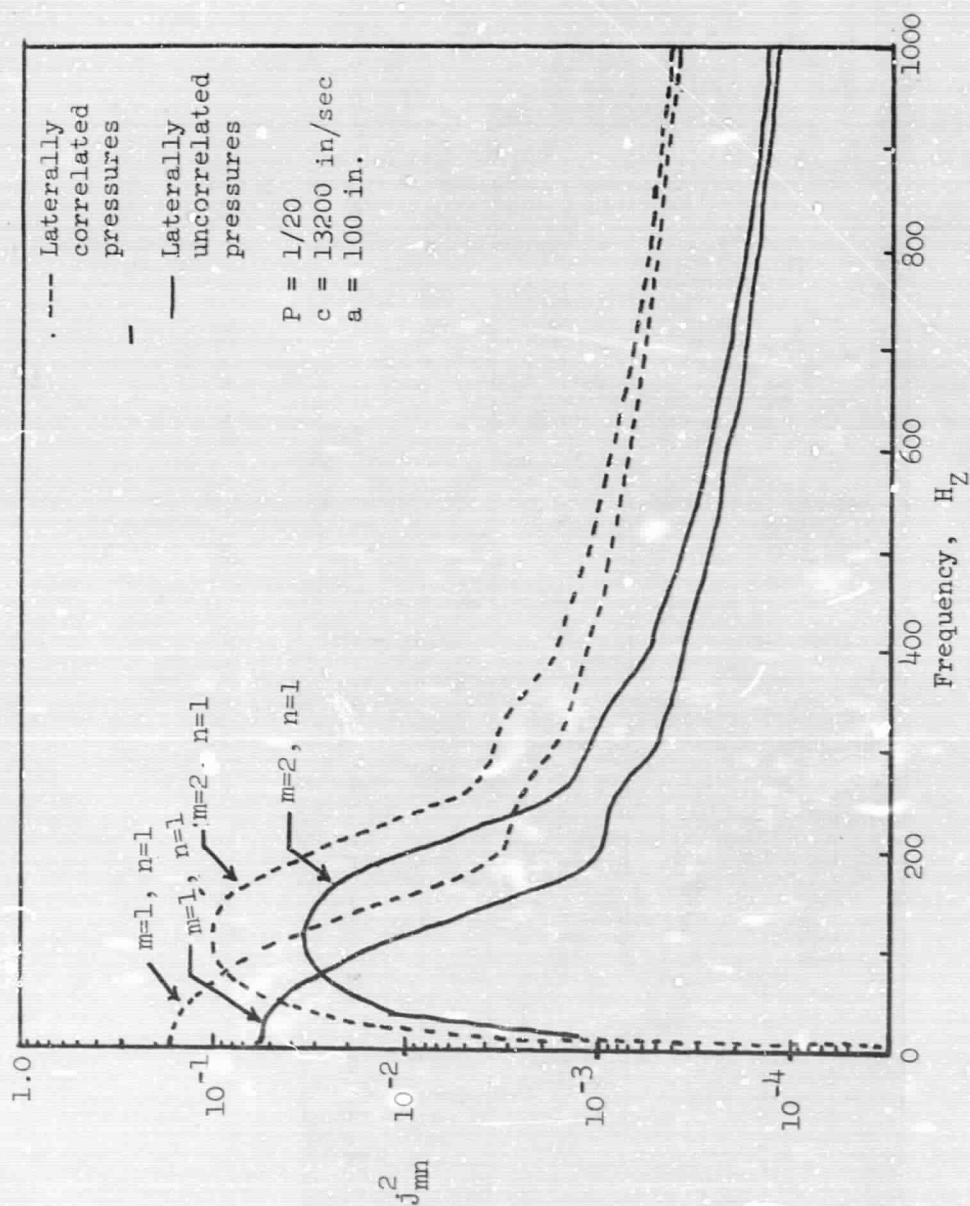


Figure 5.- Comparison of joint acceptance coefficients for laterally uncorrelated pressures with Bozich's results.

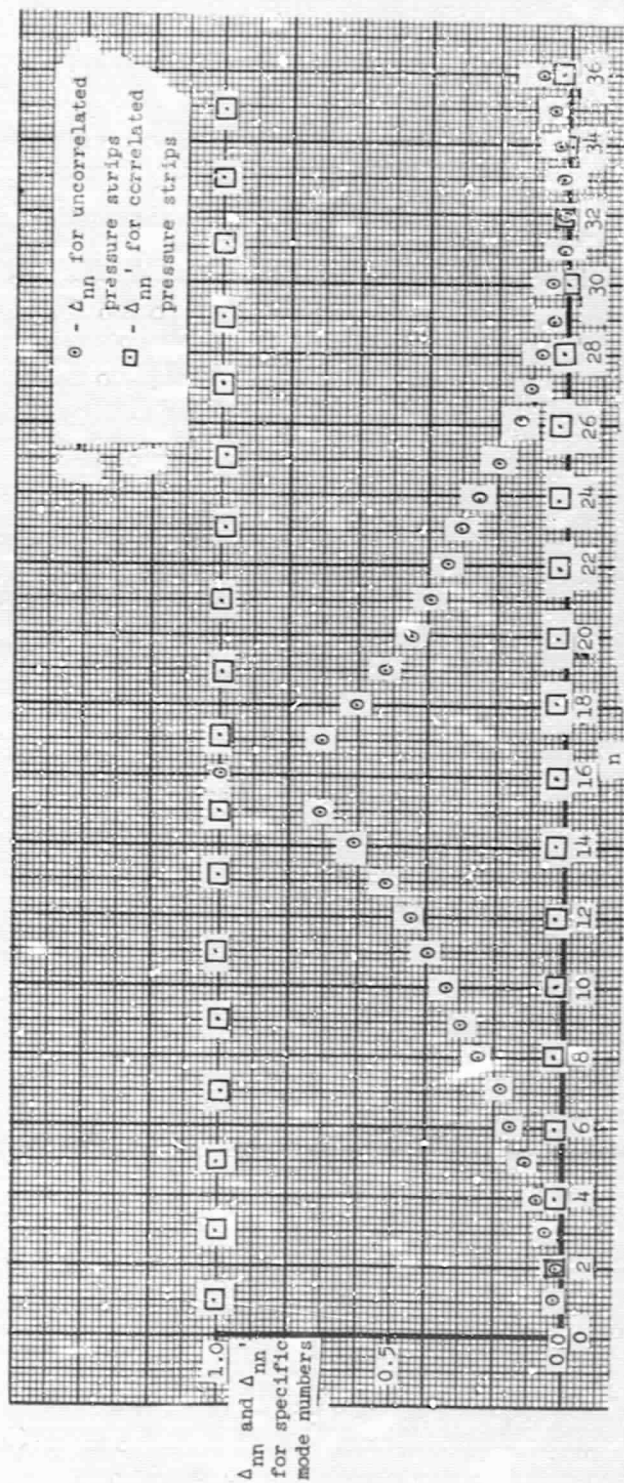


Figure 6.- Comparison of Δ_{nn} and Δ_{nn}' for $d = 16$.

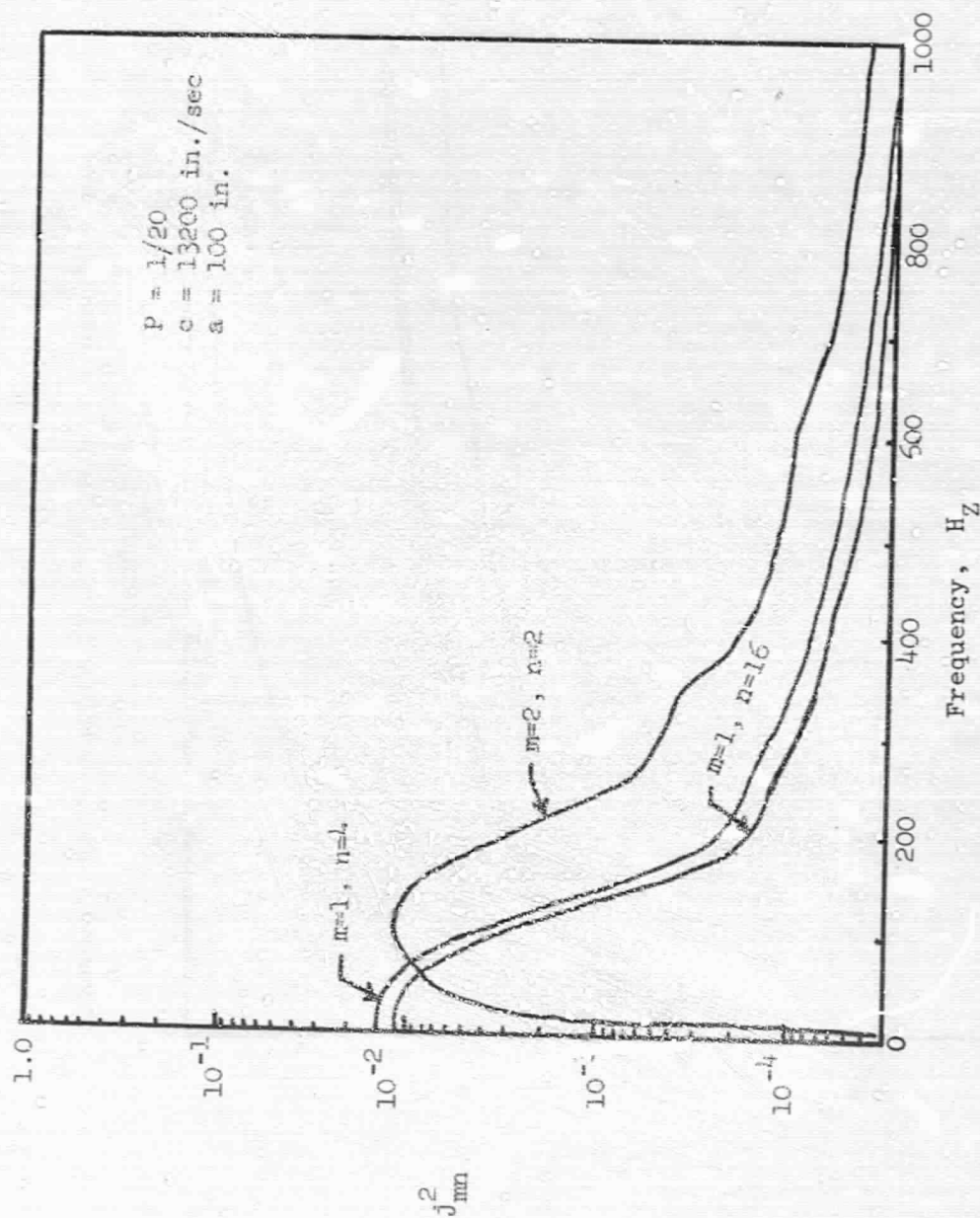


Figure 7.- Joint acceptance coefficients for a decay factor of $1/20$.

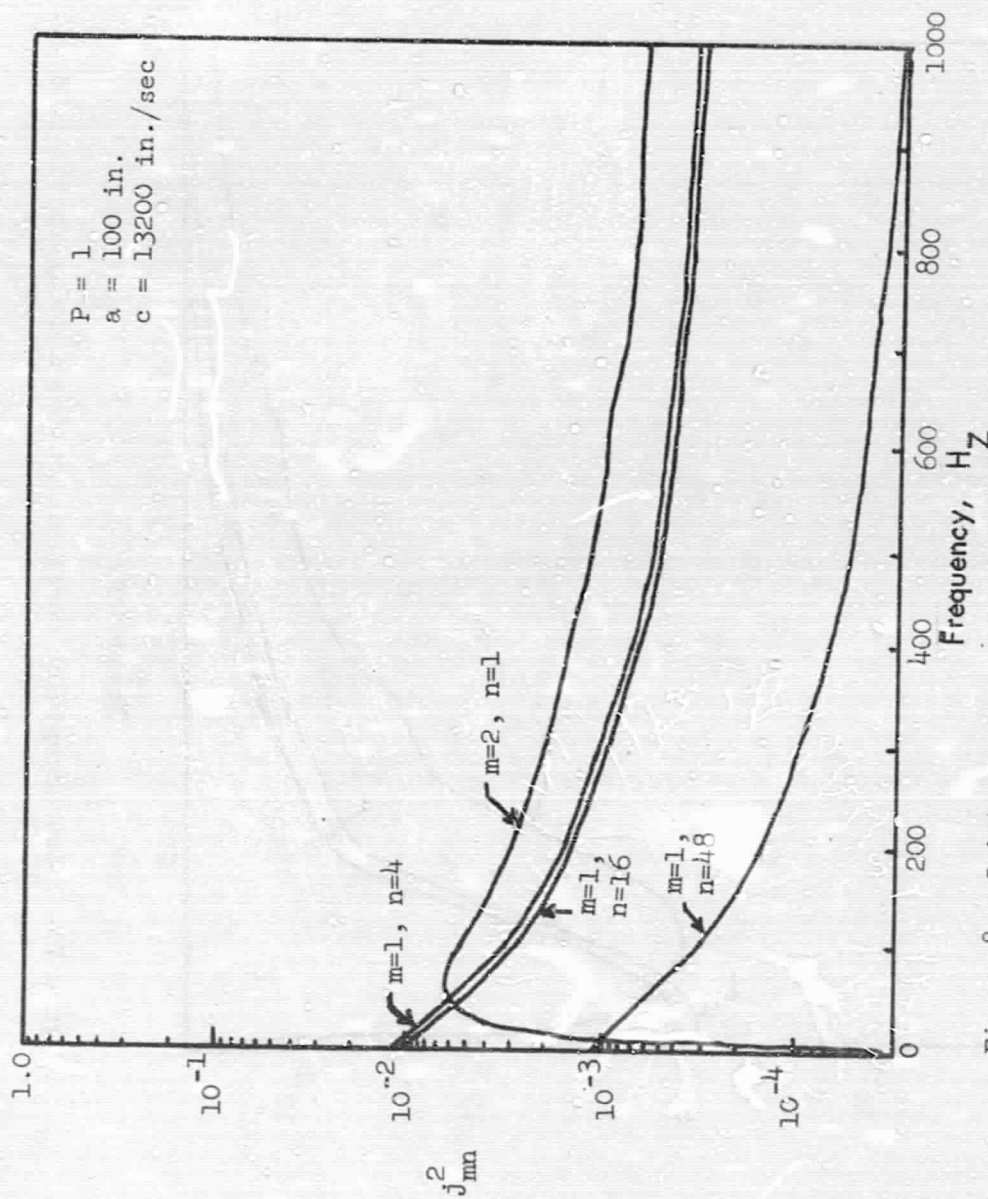


Figure 8.- Joint acceptance coefficients for decay factor $P = 1$.